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ABSTRACT

In earlier research the writer was constrained by the variables which were continuously appearing in both experimental class teaching experiences and various psychological experiments to distinguish four levels of cognitive sophistication in so far as mathematical material was concerned between the ages of 7 years and 17 years. In current research the investigator has found that similar parameters can be used to distinguish the same four levels of cognitive functioning in school curriculum content areas vastly different from mathematics e.g. History, Geography, Commerce, Modern Languages etc. In the first section, four of these parameters or cognitive variables which are apparently consistent across content areas have been selected and summarized. The second section describes a phenomenon recurring in the cognitive studies carried out by the writer, a phenomenon apparently connected with the space available in the individual's working memory. A tentative explanation is offered which is not incompatible with the information processing thoeries of Juan Pascual-Leone and Robbie Case. (Author)

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STAGES IN THINKING AND SCHOOL CURRICULUM CONTENT

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Stages in Thinking and School Curriculum Content.

In earlier research the writer was constrained by the variables which were continuously appearing in both experimental class teaching experiences and various psychological experiments (vide, Collis, 1971, 1972, 1973, 1974, 1975 (a), 1975 (b), 1976) to distinguish four levels of cognitive sophistication* in so far as mathematical material was concerned between the ages 7 years and 17 years. In current research (Collis & Biggs 1977) the investigators have found that similar parameters can be used to distinguish the same four levels of cognitive functioning in school curriculum content areas vastly different from mathematics e.g. History, Geography, Commerce, Modern Languages etc.

Four of these parameters or cognitive variableswhich are apparently consistent across content areas have been selected and summarised below for each of the four stages under consideration and in three separate Universes of Discourse.

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* The four stages which emerge may be called early concrete-operational (7-9 years)**, middle concrete-operational (10-12 years), late concrete-operational (or concrete-generalization) (13-15 years) and formal-operational (16+ years). These stages correspond, in operational terms, to Piaget's stages IIA, IIB, IIIA and IIIB. (Inhelder & Piaget, 1958).

** Age ranges are given to orient the reader to the approximate size and grade level of the child - they are not by any criteria to be considered as definitive.

EARLY CONCRETE OPERATIONAL

STAGE IIA

Cognitive Variable	General Material	Mathematical Material	Judgmental Material
Data dimension and Closure	Selects one relevant dimension and closes on it.	One operation and must be immediately closed.	Selects one perceptually dominating aspect and makes a decision on this basis alone.
Necessity for Consistency	No requirement.	No requirement.	No requirement.
Interacting Systems	Does not recognize a closed system and will interfere with it from the outside without realizing it.	No basis for deciding on equivalent statements (e.g. 3+2 = or ≠ 4+1) other than by closure.	Does not recognize the significance of interactions but selects one aspect or part of the whole.
Inverse	Only notion is physical e.g. what is hidden by wrapping can be discovered by unwrapping.	Operation of subtraction related directly to physical "taking away".	Counter examples in data ignored.

MIDDLE CONCRETE OPERATIONAL

STAGE IIB

Cognitive Variable	General Material	Mathematical Material	Judgmental Material
Data dimension and Closure	Closes on the basis of two or more relevant (but not necessarily related) aspects.	Two or more operations but closed in sequence.	Selects two or more relevant aspects and uses them as a series of non-interacting reasons for the decision.
Necessity for Consistency	Intuitively seeks consistency in dealings with material.	Prefers to work with tables and operations which give consistent results but cannot give logical reason for preference.	Likes judgments made from same data to be consistent.
Interacting Systems	Able to work within a simple concretely based single system of co-variation.	Concretely based equivalent statements can be manipulated without recourse to closure e.g. 3+2 = 4+1 by rearrangement of cardinal numbers of sets represented in statement.	Makes decision on basis of 2 or more aspects of the data which may be seen as co-varying with the decision in terms of a simple concrete relationship.
Inverse	Seen as complete and irreversible negation.	Irreversible negation e.g. subtration "destroys" addition.	Makes decision by selecting two or more propositions. When both pro and contra statements appear, decides on basis of one and ignores the other - does not note possible interactions.

STAGE IIIA

Cognitive Variable	General Material	Mathematical Material	Judgmental Material
Data dimension and Closure	Closes on the basis of all dimensions (and relationships between them). When conclusion is a generalisation it is based only on the specific data given and is thus often inadequate.	Requires guarantee that elements and their combinations can be verified in reality i.e. that an operation may be closed at any time during a sequence and give a unique result. Thus can work on what appear to be abstractions.	Makes a decision after using all the facts and their relationships and interactions as given in the data.
Necessity for Consistency	Requires consistency in his dealings with the data within the limits imposed by his concrete perception of them.	Requires consistency within concretely based simple systems.	Expects judgments to be consistent if made on basis of similar concrete data.
Interacting Systems	Able to work with two concretely based simple systems separately and then relate the results.	Able to relate formula statements such as V = LxBxH to changing concrete values but unable to treat elements of formula as interrelated variables.	Reasons on the basis of several aspects of the data (and their combinations) interacting within one or two relatively simple concrete systems. Results of interaction can be brought together for concrete decision.
Inverse	Seen as a reversable negation.	Reversible negation, e.g. subtraction "undoes" addition which can be "done up" by a reverse process.	When both pro and contra statements present themselves in a simple concrete system can take account of possible interaction in making a judgment by combining both aspects in a compound concretely based statement.

FORMAL OPERATIONAL

STAGE IIIB

Cognitive Variable	General Material	Mathematical Material	Judgmental Material
Data dimension and Closure	Able to go beyond specifics in data. Uses data to make inferences and test hypothesis. Holds question open while considering all dimensions and interactions both within and outside data then makes a closure, qualified if appropriate.	Does not need to relate elements, operation or their combinations to physical analogues but is able to regard a well-defined abstract system with its definitions relationships and rules as the reality. Closure is not attempted until all possibilities are tested.	Goes beyond immediate facts and relationships within data to relevant principles suggested both by the data and by the nature of the judgment required. Makes a qualified judgment if appropriate.
Necessity for Consistency	Can recognize inconsistencies and can resolve apparent inconsistencies.	Requires consistency in any system of operations set up (simple or complex; concrete or abstract). Can recognize and analyse latent inconsistencies in a complex system.	Expectation of consistency in judgments on complex issues within a broad abstract framework.
Interacting Systems	Able to work with two or more abstract systems and see the relationships involved.	Treats various mathematical statements as a series of interrelated relationships between variables which he can manipulate by operating within the rules inhering in the system as defined.	Can make judgments which involve the interaction between two or more complex systems. Judgment may be qualified and relate to abstract principles.
Inverse	Seen as a reciprocal process - one which balances or compensates for an earlier action.	Seen as working directly on the operations themselves in such a way as to balance or compensate without necessarily affecting the existence of an earlier operation or upsetting the original relationships existing in the statement.	Pro and contra statements are balanced one against the other and the evidence weighed.

Summary of Characteristics of Reasoning at each Stage

Perhaps a summary of what has been suggested so far would assist to clarify the main points being made.

If a subject is asked to consider a question in which data, upon which a conclusion can be based, is given, then the following model might apply.

If a, b, c, d ... etc. are propositions given in the data and related to physically or iconically available elements and operations (e.g. 3 + 5; King William devasted the North);
and .... x, y, z are propositions, questions, ideas from outside the data, but suggested as worth consideration by them (e.g. Is it always so? What if...? etc.);
and α, β, α^1, β^1 etc. are possible conclusions (α & β represent clear-cut conclusions from the data; α^1, β^1 represent qualified conclusions) then the characteristics of the four stages might be summarised as below.

Early Concrete Operational (Stage IIA)

Conclusion: α or β

Reasoning: One of a, b, c or d etc.

Middle Concrete Operational (Stage IIB)

Conclusion: α or β

Reasoning: Two or more of a, b, c or d etc. as sequence of disjoint statements.

Concrete Generalisation (Stage IIIA)

Conclusion: α or β

Reasoning: Two or more of a, b, c or d etc.; some combinations of or interactions between a & c, or b & c, b & c & d etc. All statements given consideration either alone or in some combination. Combinations and/or interactions considered not exhaustive.

Formal Operational (Stage IIIB)

Reasoning: .. x, y, z brought into consideration. a, b, c, d ... etc. taken in all possible combinations/interactions (1 at time, 2 at time etc.) and linked with x, y, z as appropriate.

Conclusion: α, β, α^1, β^1 as appropriate.

Stages of Thinking and Information Processing

Thus far the paper describes some typical features of reasoning at the various cognitive functioning levels without helping us to understand the cognitive mechanism which gives rise to these levels. However, a useful approach to this latter problem may be through an investigation of a phenomenon which continues to recur in the writer's studies of children's thinking. The remainder of this paper will be devoted to a description of the phenomenon followed by what appears to be a testable explanation of the phenomenon in terms of information processing capacity.

The Phenomenon

A Stage IIA Example

One study (Collis, 1975a) showed that stage IIA children were clearly capable of working meaningfully with elementary mathematical items which involved two elements and one operation, (e.g. 2 + 3) but were unable to work successfully when a further element and operation were introduced (e.g. 2 + 3 + 4). Success with the latter type of item came with children at the next stage (i.e. stage IIB). When tested individually the stage IIA children seemed to be unable to retain all the necessary information long enough to process it.

A typical response to the problem would go like this :

E : What number does 2 + 3 + 4 equal?

S : 2 + 3 = 5 and (pause) what was the other number?

E : 4

S : Ah yes! Now, 2 plus (pause) what is the sum again?

A Stage IIB Example

This example comes from a video-taped interview by the writer. The subject is an eleven year-old girl.

E : You are given that

$$\left.\begin{array}{l} y = 365 \\ \text{and} \\ y + 489 = 489 + 365 \end{array}\right\} \text{ written on chalk board}$$

Can you tell me the value of "y"?

S : I think so - may I work on the board?

E : Of course (offering chalk).

(S was encouraged to talk aloud as she thought and worked). S moved to the board and indicated that she was going "to add up 489 and 365." She did, thus,

$$\begin{array}{r} 489 \\ 365 \\ \hline 854 \end{array}$$

Then she came back to the statement $y + 489 \doteq 489 + 365$ and, with furrowed brow, read the whole statement aloud as though trying to impress it on her memory. At the same time she gave occasional glances at the working she had already done.

Suddenly she exclaimed "ah" and moved quickly to where she had her previous working, chalk poised, to begin work again. By the time she reached there (a fraction of a second) she had forgotten what she was about to do and so returned to reading the expression again. After some more shuffling back and forth she turned to E and said, in a resigned voice, "I can't do it." All the time S exhibited outward signs of great mental effort and, before giving up, very real stress.

In this case also the problem seemed to be associated with an inability to retain all the necessary information long enough to process it.

Children interviewed who were at the stage IIIA level clearly had no problem with this exercise.

A Stage IIIA Example

Another study (Collis, 1973) set up an elementary mathematical system and asked the subjects to draw conclusions within the constraints of the system. The phenomenon under discussion became apparent at stages IIIA but was resolved by subjects at the next stage.

This particular study defined an operation "*" such that a * b = a + 2 x B where a, b, c etc. were zero and the positive integers (0, 1, 2, ...). The subjects were asked to deduce when statements such as the following would be true.

(i) a * b = b * a

(ii) a * (b * c) = (a * b) * c

(iii) a * x = a

The typical approach for stage IIIA subjects was to work by substituting "+" and "x" for the "*" operation in items (i) and (ii) (thus ignoring the given rules) but on reaching item (iii) seemed to realize that this procedure was not adequate. They then endeavoured to bring in the given definition of the operation. However, they seemed to run into the problem of remembering what stage the calculation was at and worked somewhat like this :

a * x = a

a + x = a (S decides not satisfactory and refers back to instructions and repeats a * b = a + 2 x b).

a + = a (S, speaking aloud, decides he wants something to replace "x").

Reference is now made back and forth between

a * b = a + 2 x b and a + = a

Occasionally in this process he seems to have a sudden insight but it always disappears before he can come to grips with the problem.

Finally writes something like,

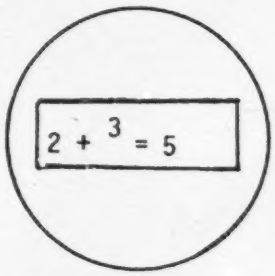
"a + 2 x b = a + 2 x b

statement is true when x = 2 x b"

Again the subject typically exhibits signs of cognitive strain and his oral repetition of parts of the problem seem to indicate the difficulty he has in bringing together the relevant data and processing it at the same time.

An Information Processing Explanation

In all cases protocols for individual subjects exhibiting this phenomenon seem to show that S behaves as if he is exceeding his particular processing capacity. Let us take the stage IIA example given above. Suppose, in the diagram below the rectangle represents the space available for processing data. The space is sufficient to take in two elements and one operation and perform the necessary calculation.



However, the space is fully occupied. If one now introduces another operation and element the space available is exceeded and overflow results. Thus part of the data necessary for a successful outcome is forced out of consideration. As the subject realizes the situation and retrieves one piece of data another piece is forced out of the space and so on.

This kind of explanation fits all the aspects of situation as observed by the writer; the oral repetition of the data, the continual refocussing on different parts of the data, the physical symptoms of strain and confusion etc.

This "explanation" of course has yet to be set up in a formal experiment and tested. It would, however, appear to be an extremely valuable hypothesis to test. Whether it were supported or not, it would add considerably to our knowledge on the phenomenon which is the subject of these symposia while, at the same time, throwing some more light on the possible link between two important theories of cognition.

References

Collis, K.F. "Concrete Operational Thinking and Formal Operational Thinking in Mathematics". The Australian Mathematics Teacher. 25, 3, 77-84, 1969.

Collis, K.F. "A Study of Concrete and Formal Reasoning in School Mathematics". Australian Journal of Psychology. 23, 3, 289-296, 1971.

Collis K.F. A Study of Concrete and Formal Operations in School Mathematics. Ph.D. Thesis, University of Newcastle, New South Wales, 1972.

Collis, K.F. "A Study of Children's Ability to Work with Elementary Mathematical Systems". Australian Journal of Psychology. 25, 2, 121-130, 1973.

Collis, K.F. "The Development of and Preference for Logical Consistency in Mathematics Material". Child Development. 45, 978-983, 1974.

Collis, K.F. A Study of Concrete and Formal Operations in School Mathematics : A Piagetian viewpoint. Melbourne, A.C.E.R., 1975(a).

Collis, K.F. The Development of Formal Reasoning. University of Newcastle, New South Wales, 1975(b).

Collis, K.F. "Mathematical Thinking in Children". In Varma, Ved P. & Williams, Phillip (Eds.) Piaget, Psychology and Education. London, Hodder & Stoughton, 144-154, 1976.

Collis, K.F. & Biggs, J.B. Classroom Examples of Cognitive Development Phenomena. Report in Preparation for Education Research and Development Committee, 1977.

Inhelder, B. & Piaget, J. The Growth of Logical Thinking from Childhood to Adolescence. London, Routledge & Kegan Paul, 1958.